Some Reasons to Formalize “If…, then…” with the Connective ⊃

We have defined ⊃ in terms of its truth table in the following way:

p	q	p ⊃q
T	T	T
T	F	F
F	T	T
F	F	T

The Problem:

We have also seen that this symbol is supposed to represent formally the phrase from ordinary language “If…, then…”.

p	q	If p, then q
T	T	T
T	F	F
F	T	T
F	F	T

There immediately arises a problem; while the second row of this truth table seems natural enough, the other three are far from it! For one thing, the statement “If p, then q” doesn’t seem to say anything at all about what will be the case if p does not happen. So why should we fill in values of “T(rue)” for those rows in which p is false?

Secondly, it might seem right at first that “If p, then q” is true when p and q are both true. But is this really the case? We can make an “If…, then…” statement out of any two propositions. So let’s pick “the moon is larger than a raisin” and “Professor Schupbach has never traveled to Spain.” Both of these are true, but what about the compound proposition “If the moon is larger than a raisin, then Professor Schupbach has never traveled to Spain”? That statement is odd at best and false at worst! So is it really the case that a conditional statement is true whenever its antecedent and consequent are both true?

Why the mismatch here between the formal language and everyday language? What exactly is the problem? Is there any good reason to define “If…, then…” symbolically in this way? In other words, why should we think we can formalize “If…, then…” with the symbol ⊃?!

Responses:

(1) *Falsifiability*: The first reason is actually less of a reason than a convenient and (I think) helpful way to think about “if..., then...” statements. It is helpful insofar as it makes the move of formalizing conditionals in terms of the symbol ⊃ more intuitively palatable. Often times, when

we use "If…, then…" sentences, we are saying something about the way that we can falsify the expressed statement. Consider the sentence "If I go to sleep now, I will be awake in time for the big game." What would it mean to show that this statement is false? To show that it's false, I would have to go to sleep now, and still be sleeping at the time of the big game! In other words, for the statement to be shown false, its antecedent must be true at the same time that its consequent is false. In any other case (i.e., whenever the antecedent and consequent is true, and whenever the antecedent is just false), we can do nothing to show that the conditional is false. Because this is the only clear way in which we can imagine verifying that the proposition is false, in all other cases we might assume that its true.

This is essentially the argument given in Patrick Hurley's popular textbook *A Concise Introduction to Logic*. Here is the relevant passage:

> *Imagine that your instructor made the following statement: "If you get an A on the final exam, then you will get an A for the course." Under what conditions would you say that your instructor had lied to you [told you something false]? Clearly, if you got an A on the final exam but did not get an A for the course, you would say that she had lied. This outcome corresponds to a true antecedent and a false consequent. On the other hand, if you got an A on the final exam and also got an A for the course, you would say that she had told the truth (true antecedent, true consequent). But what if you failed to get an A on the final exam? Two alternatives are then possible: Either you got an A for the course anyway (false antecedent, true consequent) or you did not get an A for the course (false antecedent, false consequent). In neither case, though, would you say that your instructor had lied to you. Giving her the benefit of the doubt, you would say that she had told the truth.*

(2) *Synonymy with "It cannot be the case that p is true and q is false"*: We can try to figure out the truth table definition of "If p, then q" by first looking for another way of phrasing that same statement. The statement "If p, then q" implies the statement "p cannot be true at the same time that q is false". Moreover, we might note that whenever we say "p cannot be true at the same time that q is false" we are implying that "if p [holds true], then q [must hold true too]" (because otherwise p would be true at the same time that q is false." So these just seem to be two ways of saying the same thing – they imply each other.

But if they are just different ways of saying the same thing, then we should be able to clarify the truth table definition of "If p, then q" (or, symbolically, p⊃q) by looking at the truth table for "p cannot be true at the same time that q is false." This latter sentence can be formalized straightforwardly as ~(p&~q), and we can work out the truth table for this formal statement as follows:

p	q	~q	p & ~q	~(p & ~q)
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

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Note that the truth values under ~(p&~q) are T, F, T, and T. And so these are the values that we should write in under p⊃q. But that gives us the truth table at the top of the first page of this handout! [This is essentially the same argument given in your own textbook by Sinnott-Armstrong and Fogelin on p. 164.]

(3) *Preserving intuitive properties of the conditional*: It seems plausible to think that the following things are true of conditionalization:

- *Modus Ponens*: The argument from “p” and “If p, then q” to “q” is valid
- *Affirming the consequent*: The argument from “q” and “If p, then q” to “p” is invalid
- All statements of the form “If p, then p” are logical truths

But the only binary connective (which takes a complete truth-table) that preserves these intuitive properties is that which we defined as ⊃. In other words, if you wanted to associate the everyday language phrase “If…, then…” with any other such connective, you would have to deny at least one of the above three intuitive properties.

One other way to phrase this is as follows: sure, filling in some of the cells of our truth table for ⊃ in the way that we do seems strange. However, if we go ahead and do it anyway, then we find that ⊃ behaves well in accordance with our intuitions about the ordinary language phrase “If…, then…”.